UNIVERSITY OF SASKATCHEWAN
GE125.3 - Engineering Mechanics II
MIDTERM EXAMINATION

TIME: 2 HOURS **February 11, 2005**

INSTRUCTIONS:

1. Answer ALL questions. **All question carry equal marks**.
2. Only calculators, pens, pencils, and drawing aids are allowed for the exam.
3. Show your solution(s) in the space below the question. You may also write on the reverse side (if you need more space).
4. Make sure you supply your **Name**, **Student Number**, **Section Number**, and **Examination Room** in the space provided below. Also, place your name at the top of each sheet. ***You will be penalized for failing to do so***.

NAME: ____
First Name **Last Name**

STUDENT NUMBER: ____________________________

SECTION NUMBER: ____________________________

EXAMINATION ROOM: ____________________________

MARKS

1. ______________/25

2. ______________/25

3. ______________/25

4. ______________/25

TOTAL:____________**/100**

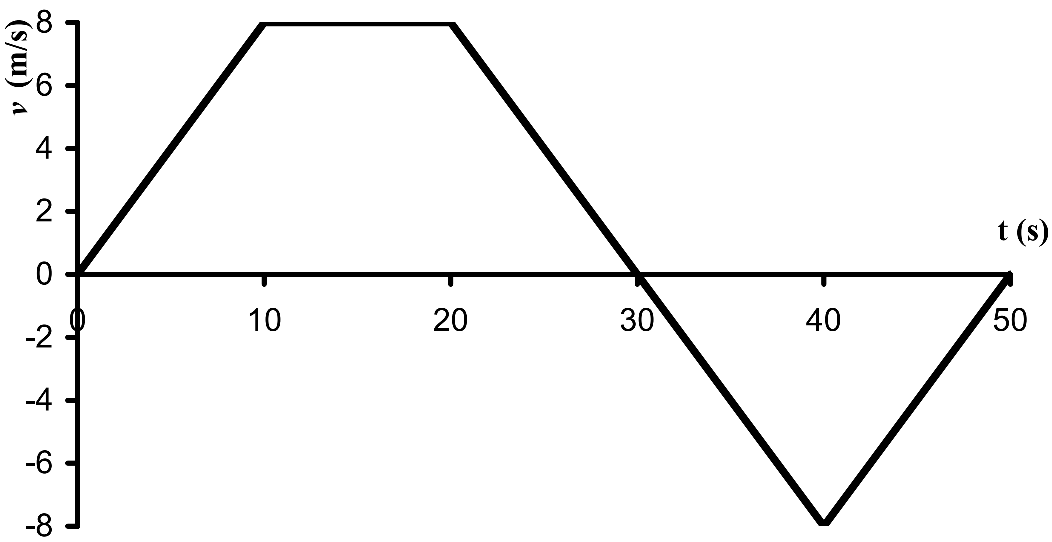
EXAMINATION ROOM LOCATIONS:

Section 02 (11:30 a.m. - 12:20 p.m.):
THORV 105 (A - Lof)
THORV 271 (Lor - Zz)

Section 04 (02:30 p.m. - 03:20 p.m.):
BIOL 106 (A - Liu)
ENG 1B71 (Lo - Zz)

Student Name: ___________________________________

1. A go-cart travels along a track with the velocity shown in the v-t graph. Construct the s-t and a-t graphs for the motion and write the equations for the a-t and s-t curves. What is the average speed during the 50 seconds the go-cart has travelled?

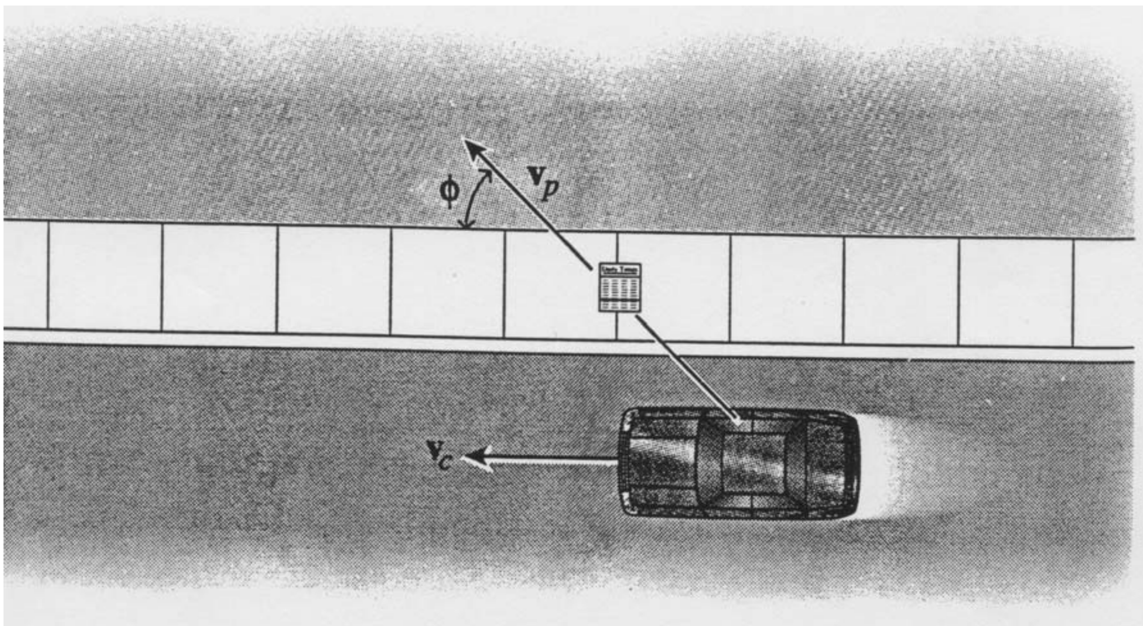


Student Name: ___________________________________

2. A man distributing newspapers by car tosses a bundle of papers from the car as shown below. If the car is traveling at 15 km/h and the papers are tossed with a velocity of 5 m/s relative to the car and perpendicular to the motion of the car, determine:

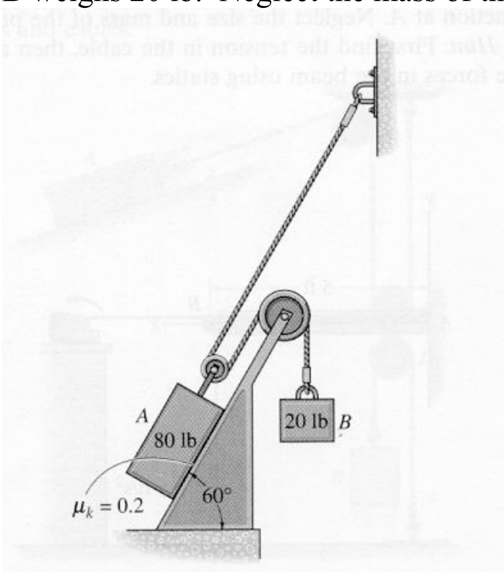
(a) The velocity $\mathbf{v_P}$ of the papers relative to the sidewalk.

(b) The angle ϕ between the velocities $\mathbf{v_P}$ and $\mathbf{v_C}$.



Student Name: ___________________________________

3. Determine the acceleration of Block A when the system is released. The angle of incline is 60^{o} from horizontal. The coefficient of kinetic friction is 0.2. Block A weighs 80 lb and Block B weighs 20 lb. Neglect the mass of the pulleys and cord.



Student Name: ________________________________

4. A particle ***P*** moves along the curve $y = (x^2 - 4)$ m with a constant speed of 5 m/s. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

Student Name: __________________________________

Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

Variable a	*Constant $a = a_c$*
$a = \dfrac{dv}{dt}$	$v = v_0 + a_c t$
$v = \dfrac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$$v = \dot{s} \qquad a_t = \dot{v} = v\frac{dv}{ds}$$

$$a_n = \frac{v^2}{\rho} \qquad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant $\alpha = \alpha_c$*
$\alpha = \dfrac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \dfrac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r \qquad v = \omega r \qquad a_t = \alpha r \qquad a_n = \omega^2 r$

Relative General Plane Motion—Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion—Trans. and Rot. Axis

$$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

KINETICS

Mass Moment of Inertia $I = \int r^2\,dm$

Parallel-Axis Theorem $I = I_G + md^2$

Radius of Gyration $k = \sqrt{\dfrac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1\text{-}2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \dfrac{dU}{dt} = \mathbf{F}\cdot\mathbf{v} \qquad \epsilon = \dfrac{P_{\text{out}}}{P_{\text{in}}} = \dfrac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int \mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int \mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

MATHEMATICAL EXPRESSIONS

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Hyperbolic Functions

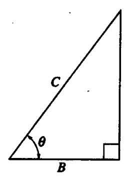
$\sinh x = \dfrac{e^x - e^{-x}}{2}$, $\quad \cosh x = \dfrac{e^x + e^{-x}}{2}$, $\quad \tanh x = \dfrac{\sinh x}{\cosh x}$

Trigonometric Identities

$\sin\theta = \dfrac{A}{C}$, $\quad \csc\theta = \dfrac{C}{A}$

$\cos\theta = \dfrac{B}{C}$, $\quad \sec\theta = \dfrac{C}{B}$

$\tan\theta = \dfrac{A}{B}$, $\quad \cot\theta = \dfrac{B}{A}$



$\sin^2\theta + \cos^2\theta = 1$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos\theta = \pm\sqrt{\dfrac{1+\cos 2\theta}{2}}$, $\quad \sin\theta = \pm\sqrt{\dfrac{1-\cos 2\theta}{2}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

Power-Series Expansions

$\sin x = x - \dfrac{x^3}{3!} + \cdots \qquad \sinh x = x + \dfrac{x^3}{3!} + \cdots$

$\cos x = 1 - \dfrac{x^2}{2!} + \cdots \qquad \cosh x = 1 + \dfrac{x^2}{2!} + \cdots$

Derivatives

$\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{du}{dx}$

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{d}{dx}(\cot u) = -\csc^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sec u) = \tan u\sec u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\csc u) = -\csc u\cot u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sin u) = \cos u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cos u) = -\sin u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\tan u) = \sec^2 u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\sinh u) = \cosh u\dfrac{du}{dx}$

$\dfrac{d}{dx}(\cosh u) = \sinh u\dfrac{du}{dx}$

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INTEGRALS

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) + C$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ba}}\ln\left[\frac{a+x\sqrt{-ab}}{a-x\sqrt{-ab}}\right] + C, \quad ab<0$$

$$\int \frac{x\,dx}{a+bx^2} = \frac{1}{2b}\ln(bx^2+a) + C,$$

$$\int \frac{x^2\,dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b\sqrt{ab}}\tan^{-1}\frac{x\sqrt{ab}}{a} + C, \quad ab>0$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left[\frac{a+x}{a-x}\right] + C, \quad a^2>x^2$$

$$\int \sqrt{a+bx}\,dx = \frac{2}{3b}\sqrt{(a+bx)^3} + C$$

$$\int x\sqrt{a+bx}\,dx = \frac{-2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$$

$$\int x^2\sqrt{a+bx}\,dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right] + C, \quad a>0$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C$$

$$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3} + \frac{a^2}{8}\left(x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}\right) + C, \quad a>0$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right] + C$$

$$\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2 \pm a^2)^3} + C$$

$$\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln\left(x + \sqrt{x^2 \pm a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}}\ln\left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}}\right] + C, \quad c>0$$

$$= \frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{-2cx-b}{\sqrt{b^2-4ac}}\right) + C, \quad c>0$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax) + C$$

$$\int x^2\cos(ax)\,dx = \frac{2x}{a^2}\cos(ax) + \frac{a^2x^2-2}{a^3}\sin(ax) + C$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int x e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) + C$$

$$\int \sinh x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \sinh x + C$$